BENTON HARBOR POWER PLANT LIMNOLOGICAL STUDIES

PART XX. STATISTICAL POWER OF A PROPOSED METHOD FOR DETECTING THE EFFECT OF WASTE HEAT ON BENTHOS POPULATIONS

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ABSTRACT

In Part 18 of this report series, a five-way analysis of variance was proposed to assess the effect of waste heat on benthos populations near the Cook Plant. The complete period of study would be 1971 through 1978. Ultimately, four years of preoperational and four years of operational data would be available for the analysis. This report calculates the expected sensitivity of the proposed ANOVA design.

An equation given by Sokal and Rohlf (1969) can be used to express the least detectable true difference between two treatment means as a function of the following: the significance level of the test, the statistical power (the desired probability that a true difference will be detected), the degrees of freedom for error, the number of observations per treatment, and the error standard deviation. In this report the significance level is set at 5% and the power at 95%. These requirements lead to an estimate of the smallest true change in the benthos that is detectable by the existing sampling program. This is expressed as a change due to plant operation in the ratio of the true mean population at the inner stations (near the outfall) to the mean at the outer stations. It turns out that this ratio would have to increase or decrease by a factor of 5.48 for a change to be detected by the planned ANOVA.

An important step in the calculation is to find an estimate $\hat{\sigma}$ for the error standard deviation σ. This will be the square root of the mean square used as the denominator for the F-test. In the present design, which is a mixed model nested ANOVA, the test for a heat effect is the test of the interaction of the construction time and outfall distance factors. The appropriate denominator can be found using a table of expected mean squares. The square root of this denominator gives $\sqrt{\sigma_\epsilon^2 + 27\sigma_{\beta\gamma}^2}$ as the value for σ, where σ_ϵ^2 is the within-cell error (sampling error) and $\sigma_{\beta\gamma}^2$ is the variance of

the model term for the interaction of year and outfall distance. Data already taken are used to obtain the following estimates: $\hat{\sigma}_{\varepsilon}^{2} = .49527$, and $\hat{\sigma}_{\beta\gamma}^{2} = .0100($

Two features of this model are distinctive: (1) the nesting of the year factor, and (2) the use of a non-zero estimate of the interaction variance $\sigma_{\beta\gamma}^{2}$. While these choices reduce the predicted power, they probably lead to a more realistic model.

INTRODUCTION

In a previous report in this series (Johnston 1973), two methods were proposed for assessing the effect of waste heat on benthos populations near the Cook Plant. The first of these was a graphical technique. The second was a five-way analysis of variance (ANOVA) that would require four years of pre-operational data and four years of operational data. The present report is concerned with the second method. It will give a calculation of the least change in the benthos that is detectable with 95% power by the proposed analysis of variance.

METHOD AND RESULTS

Significance and Power

In this report, the level of significance α used will be .05 in the statistical test for the presence of a heat effect. This means that we accept a 5% probability of a Type I error. Such an error would be a conclusion that a certain biological parameter had changed when there had in fact been no change. In the analysis of variance there is also a risk of a Type II error, that is, that a true change in the biological parameter would not be found significant by the test. The probability of a Type II error is β, and the statistical power P is equal to 1-β. In general, for a given level of α and a given true change, the power of a test increases with the number of replicates. In this report we will require that P =.95 and compute the true change in the benthos population for which this power will be achieved. An equation relating these quantities is as follows (derived from an equation of Sokal and Rohlf 1969, p. 247):

$$\delta = \sigma\sqrt{\frac{2}{n}}\left(t_{\alpha[\nu]} + t_{2(1-P)[\nu]}\right)$$

where

δ = least detectable true difference

σ = true error standard deviation

ν = degrees of freedom of the error mean square

n = number of observations at each of the two treatment levels

t = Student's t

α = significance level

P = power (the desired probability that a difference will be found to be significant)

An example to which this formula could be applied is the following. The means of two treatments are to be compared where there are n observations per treatment and σ is the true standard deviation of the observations about their respective treatment means. The test could be either a two-sample t-test or a one-way ANOVA. If we set α = .05 and P = .95, then δ is the minimum amount by which the true treatment means must differ if there is to be 95% probability that the means of two samples of size n will be found significantly different at the 5% level.

The next step is to relate the above formula to the eight-year experimental design proposed in Part 18 (Johnston 1973). It was stated that the statistical test for a heat effect on a given taxon would be the F-test of the interaction of the outfall distance and construction time factors. The formula stated above can still be applied, since this interaction has only a single degree of freedom. In such a case, the F-test and the t-test give equivalent results $\left(F_{\alpha(1,\nu)} = t^2_{\alpha(\nu)}\right)$. The value used for σ will be the square root of the expected error mean square of the ANOVA.

The distinction of inner and outer stations was made earlier in Part 18. The idea was used in previous work on Lake Michigan benthos, in connection with the Palisades plant (Beak Consultants 1973). It is worth restating why

such a plan may be desirable. Stations far from the plant can serve as control or reference stations. If ambient lake conditions change, then stations of equal depth near to and far from the outfall should be similarly affected. By using the difference between inner and outer stations, we should be working with a quantity that is relatively less affected by ambient lake changes. It should be noted that we use the difference between the transformed population densities ($y = \log(x+1)$) in the inner and outer areas of each depth zone, averaged over all months, zones and years. (x is the actual benthos density in animals per square meter; y is the transformed density.)

Elliott (1971) favors the use of the logarithmic transformation with benthos data. "As the variance of a bottom sample is often greater than the arithmetic mean, log (x+1) is probably the most useful transformation" (op. cit. p. 32).

Expected Mean Squares for Model A

To determine the power of the test for a heat effect it is necessary to find the appropriate denominator for the F-ratio. In a factorial ANOVA with all effects fixed, this would be a simple step since the within-cell mean square (MS) would be used as the denominator in all F-tests. The present ANOVA is different since the year factor (4 levels) is nested within the construction time factor (2 levels). The relation of these two factors is as follows:

		1971	1972	1973	1974	1975	1976	1977	1978
Construction time	Before	X	X	X	X				
	After					X	X	X	X

Data will be available only for the combinations marked with an X. This design is a mixed model, since it contains both fixed and random factors (Scheffé 1959, p. 6).

The quantity we are using in the test for a heat effect is the interaction of outfall distance and construction time. To set up the F-ratio for this test, the table of expected mean squares is needed. This is given, along with the model equation for the eight-year design ("Design A"), in Table 1. The expected mean squares were derived from the linear model using a procedure given by Kirk (1968, p. 208).

In Table 1, notice that a number of interactions have been omitted. The omissions result in a model that assumes additivity for the effects of season and of depth zone. This choice was made for simplicity and because of the difficulty of assigning an ecological interpretation to further interactions. The nine terms that have been included in the linear model are those that we expect to be non-zero based on our general understanding of the system. To include all interactions would increase the number of terms in the linear model from nine to 32 and would lead to unpleasant formulas for the expected mean squares. Some remarks of Kirk may help explain our decision:

> "The selection of an experimental design and associated linear model is largely based on an experi enter's subject-matter knowledge.... The model selected should include all sources of variation that the experimenter is interested in and that are expected to contribute significantly to the total variation. In reality, all sources of variation not specifically included in the model as treatment effects become a part of the experimental error." (Kirk 1968, p. 214.)

The term that we use to test for the heat effect is $\theta_{\alpha\gamma}$. The subscripts α and γ refer to the construction time factor and the outfall distance factor, respectively. The null hypothesis is that $\theta_{\alpha\gamma} = 0$, i.e. that the startup of the plant has no effect on the difference between the mean transformed benthos density at the inner stations and the same quantity at the outer stations. Table 1 shows that the expected mean square due to the CD interaction contains $\theta_{\alpha\gamma}$: $MS_{CD} = \sigma_{\epsilon}^2 + 108\,\theta_{\alpha\gamma}^2 + 27\,\sigma_{\beta\gamma}^2$. The expected mean square

TABLE 1. *Linear Model and Expected Mean Squares for Design A, a Nested Analysis of Variance.* The notation $\beta_{j(i)}$ indicates that the factor indexed by j is nested within the factor indexed by i. This convention is used by Kirk (1968, p. 230). Note that for depth zone and season only main effects are included. Their interactions are neglected. To allow for this the error degrees of freedom shown below have been increased by the appropriate amount.

$$X_{ijkmpq} = \mu + \alpha_i + \beta_{j(i)} + \gamma_k + \alpha\gamma_{ik} + \beta\gamma_{j(i)k} + \delta_m + \zeta_p + \varepsilon_{q(ijkmp)}$$

Factor	Greek Symbol	Name	No. of Levels	Type
C	α	Construction time	2 (before, after)	fixed
Y	β	Year	4	random (nested within C)
D	γ	Outfall distance	2 (inner, outer)	fixed
S	δ	Season	3	fixed
Z	ζ	Depth zone	3	fixed
E	ε	Error	3 (no. of replicates)	

No. of observations = 2 x 4 x 2 x 3 x 3 x 3 = 432
Total degrees of freedom = 431

Source of Variation	Degrees of Freedom	Expected Mean Square
C	1	$\sigma_\varepsilon^2 + 216\,\theta_\alpha^2 + 54\,\sigma_\beta^2$
Y	6	$\sigma_\varepsilon^2 + 54\,\sigma_\beta^2$
D	1	$\sigma_\varepsilon^2 + 216\,\theta_\gamma^2 + 27\,\sigma_{\beta\gamma}^2$
S	2	$\sigma_\varepsilon^2 + 144\,\theta_\delta^2$
Z	2	$\sigma_\varepsilon^2 + 144\,\theta_\zeta^2$
CD	1	$\sigma_\varepsilon^2 + 108\,\theta_{\alpha\gamma}^2 + 27\,\sigma_{\beta\gamma}^2$
YD	6	$\sigma_\varepsilon^2 + 27\,\sigma_{\beta\gamma}^2$
Error	412	σ_ε^2
Total	431	

(cont.)

TABLE 1 cont.

In what follows, c is the number of levels of factor C, y is the number of levels of Y, and so on.

Since construction time is a fixed factor, θ^2_α is the mean square amplitude of the effects α_i .

$$\theta^2_\alpha = \frac{\sum_{i=1}^{c} \alpha_i^2}{c-1}$$

Similarly:

$$\theta^2_{\alpha\gamma} = \frac{\sum_{i=1}^{c} \sum_{k=1}^{d} (\alpha\gamma_{ik})^2}{(c-1)(d-1)}$$

Since year is a random factor, σ^2_β is the variance of the population from which the effects $\beta_{j(i)}$ are drawn. Similarly, $\sigma^2_{\beta\gamma}$ is the variance of the population from which the interaction effects $\beta\gamma_{j(i)k}$ are drawn.

The other σ's and θ's are defined in a parallel way. The symbol σ is used whenever one of the subscripts is β , indicating that the (randor year factor is involved. Otherwise θ is used. This notation follows that of Myers (1972, p. 188).

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for YD contains two of the above three terms: $MS_{YD} = \sigma^2_\epsilon + 27\,\sigma^2_{\beta\gamma}$. The two mean squares, MS_{CD} and MS_{YD} , can thus be used in an F-ratio to test the null hypothesis that $\theta_{\alpha\gamma} = 0$.

$$F = \frac{MS_{CD}}{MS_{YD}} = \frac{\sigma^2_\epsilon + 108\,\theta^2_{\alpha\gamma} + 27\,\sigma^2_{\beta\gamma}}{\sigma^2_\epsilon + 27\,\sigma^2_{\beta\gamma}}$$

The quantity σ^2_ϵ measures the variability among replicates; the quantity $\sigma^2_{\beta\gamma}$ measures the year-to-year variability of the difference between the inner and outer populations, averaged over all months and depth zones. Consider the null hypothesis that $\theta_{\alpha\gamma} = 0$. The F-ratio then becomes $(\sigma^2_\epsilon + 27\sigma^2_{\beta\gamma})/(\sigma^2_\epsilon + 27\sigma^2_\beta$ Under the null hypothesis the F-statistic will be distributed as tabular F(1,6)

Since $\sigma^2_\epsilon + 27\,\sigma^2_{\beta\gamma}$ is used as the denominator, the quantity $\sigma = \sqrt{\sigma^2_\epsilon + 27\sigma_\beta}$

is the appropriate error standard deviation to use in the power equation. Th next step in calculating power is to get estimates for what σ_ϵ^2 and $\sigma_{\beta\gamma}^2$ are likely to be in the complete experiment, using the portion of the data that i already available.

Estimation of the Sampling Error Variance

The quantity to be determined is σ_ϵ^2, which we will also refer to as the sampling error. Table 2 shows the surveys to be used in finding variance estimates. Surveys from July 1970 through April 1972 were taken with a samplin grid that is shown in Table 3. Beginning in July 1972, a survey plan was adopted in which station locations were chosen randomly within each area of interest. Details of this procedure were given by Mozley (1973). For convenience, the calculations that follow will assume that the random survey pla will be continued for the remainder of the eight-year experimental period (1971-1978). Under this plan, four depth zones are sampled from each of thre regions. The D region is adjacent to the plant, the N region is 7 miles nort

TABLE 2. *Dates of the Surveys Analyzed in this Report.*

Year				
1970		July	September	November
1971	April	July	September	November
1972	April	July*		October*†
1973	April*			

*The starred dates had random surveys; the others had grid surveys. Mozley (1973) gives a description of the random survey; stations used in the grid survey are displayed in Table 3.

†The survey taken in late October 1972 has been grouped with the November surveys of 1970 and 1971 for purposes of analysis of variance. There was no major survey in September 1972.

TABLE 3. *The DC, NDC and SDC Stations: A Table showing their Transects and Distances from Shore.* Entries in "Transect" columns are interpreted as follows: Stations in the row marked "0.00" are on the DC transect, which leaves shore right at the plant (41° 58.5' N., 86° 34.0' E.) and runs in a westnorthwesterly direction with bearing 290°. The other transects are parallel to this one; their distances from it are indicated in the "Transect" columns in both miles and kilometers. Positive distances refer to transects north of the plant; negative distances refer to transects south of the plant. The distance of each station from shore is measured along the transect and is given here in both miles and kilometers. Metric distances have been rounded to the nearest multiple of 0.40 km.

Transect		Distance of each station from shore (km and miles)								
		km 0.00	0.40	0.80	1.20	1.60	2.00	3.60	6.40	11.20
km	mi	mi 0.00	0.25	0.50	0.75	1.00	1.25	2.25	4.00	7.00
-11.20	-7.00		SDC-7-1	SDC-7-2*			SDC-7-3	SDC-7-4*	SDC-7-5	
-6.40	-4.00	SDC-4-0	SDC-4-1	SDC-4-2*				SDC-4-3		SDC-4-4
-3.20	-2.00	SDC-2-0	SDC-2-1	SDC-2-2*			SDC-2-3		SDC-2-4*	
-1.60	-1.00	SDC-1-0	SDC-1-1		SDC-1-2			SDC-1-3*		
-0.80	-0.50	SDC-.5-0	SDC-.5-1*	SDC-.5-2			SDC-.5-3*			
-0.40	-0.25				SDC-.25-1*					
0.00	0.00		DC-1		DC-2		DC-3	DC-4	DC-5	DC-6
0.40	0.25				NDC-.25-1*					
0.80	0.50	NDC-.5-0	NDC-.5-1*	NDC-.5-2			NDC-.5-3*			
1.60	1.00	NDC-1-0	NDC-1-1		NDC-1-2			NDC-1-3*		
3.20	2.00	NDC-2-0	NDC-2-1	NDC-2-2*			NDC-2-3		NDC-2-4*	
6.40	4.00	NDC-4-0	NDC-4-1	NDC-4-2*				NDC-4-3		NDC-4-4
11.20	7.00		NDC-7-1	NDC-7-2*			NDC-7-3	NDC-7-4*	NDC-7-5	

*Stations marked with an asterisk have not been used in major surveys since May 1, 1972.

and the S region is 7 miles south. For purposes of this report, the inner stations are those in the D region and the outer stations are those in the S region. Mozley's zone 0 (0-8 m) is here designated as A, zone 1 (8-16 m) as B, and zone 2 (16-24 m) as C. Zone 3 (greater than 24 m) has not been included in this analysis because it is too far from the outfall for an inner/outer distinction to have much meaning.

It is worth explaining what is meant by "replication" in the random survey. We consider the three randomly placed stations within a given zone to be replicates, and that each one estimates the zone mean. At each station of zones B and C, three casts were made using a three-chambered grab. The contents of chamber #1 of each grab were retained and counted. The animal densities per square meter obtained in these separate casts were simply averaged together to get a single estimate of the density of animals at that station. Since three one-third grabs were taken at each station, the total area sampled was the same as what would have been obtained with a single cast of a whole grab. However, in considering any results from a whole grab, the possibility of correlation of the catches from the three chambers should be kept in mind.

The field procedure followed in zone A was different from that used in zones B and C. Three stations were taken in each zone of each region, but at each station five casts were made and five whole grabs were counted. To obtain a value for use in this report, one of the five grabs at each station was randomly selected. The advantage of this method is that the resulting effective sample in zone A is one whole grab, just as it is in zone B and zone C. Thus, data from the three zones can be combined to get a single estimate of the error variance.

This variance can be obtained using a three-way factorial ANOVA, outfall distances x depth zones x months (2 x 3 x 3), with 3 replicates in each cell. Before analysis, the density of animals at each station was transformed using $y = \log (x+1)$. See Table 4 for the results. It shows the error variance as 0.49527. The results of the usual significance tests are shown in the table, but they are not of interest for the present purpose, which is the estimation of variances. The data used for this ANOVA are shown in Table 5 under the heading "year 3."

TABLE 4. *Estimation of the Sampling Error in the Random Survey.* The method used was a three-way factorial analysis of variance of three selected random surveys. The data were total animals per meter^2, transformed through $y = \log (x+1)$.

Factors

1. Outfall distance (2): inner, outer
2. Depth zone (3) : A (0-8m), B (8-16m), C (16-24m)
3. Month (3) : July/72, Oct./72, April/73

Within each cell, three stations were used as replicates. For details see Table 5 under "year 3."

Source of variation	Degrees of freedom	Sums of squares	Mean squares	F-ratio
Outfall distances	1	.82140	.82140	1.66
Depths	2	21.60070	10.80035	21.81**
Months	2	.46030	.23015	.46
O.D. x depths	2	.03333	.01667	.03
O.D. x months	2	1.02862	.51431	1.04
Depths x months	4	2.46057	.61514	1.24
O.D. x depths x months	4	.18806	.04702	.09
Error	36	17.82963	.49527	
Total	53	44.42261		

** Significant, $p < .01$

$F_{.05(1,36)} = 4.11$, $F_{.05(2,36)} = 3.26$, $F_{.05(4.36)} = 2.63$

TABLE 5. *Data Used in the ANOVAs of Tables 4 and 8.*

Columns headed by "x" give the actual densities in animals per square meter. Columns headed by "y" give the corresponding transformed values ($y = \log(x+1)$) See Table 8 for an analysis of variance of these data. The input to that ANOVA consisted of the means shown below, one for each month-zone combination Data for Year 1 and Year 2 were taken using the grid survey. The means for those years are of the transformed densities from two selected stations. Data for Year 3 were taken using the random survey. The means for that year are means of the transformed densities from three stations. See the text for an explanation of this procedure.

At each station of the grid survey, two casts were made. The contents of the two whole grabs were physically combined before counting. At each station of the random survey for zones B and C, three casts were made using a three-chambered grab. The contents of chamber no. 1 of each cast were retained and counted. The density shown below for each station of zones B and C is the mean of the untransformed densities of the three casts. At each station of zone A of the random survey, five casts were made and five whole grabs were counted. The density shown below for each station of zone A was obtained by randomly selecting one of the five grabs.

Inner - Year 1 (grid survey)

		July/70		Nov./70		April/71	
Zone	Station	x	y	x	y	x	y
A	NDC-.5-1	139	2.1461	50	1.7076	198	2.2989
A	SDC-.5-1	226	2.3560	893	2.9513	144	2.1614
	Mean		2.2511		2.3295		2.2302
B	NDC-1-2	3,006	3.4781	1,363	3.1348	4,059	3.6085
B	SDC-1-2	4,980	3.6973	2,483	3.3952	36	1.5682
	Mean		3.5877		3.2650		2.5884
C	NDC-1-3	10,084	4.0037	4,241	3.6276	1,993	3.2997
C	SDC-1-3	5,755	3.7601	17,326	3.2387	795	2.9009
	Mean		3.8819		3.9332		3.1003

(cont.)

TABLE 5 (cont.)

Outer - Year 1 (grid survey)

Zone	Station	July/70 x	July/70 y	Nov./70 x	Nov./70 y	April/71 x	April/71 y
A	SDC-4-1	103	2.0170	16	1.2304	18	1.2788
A	SDC-7-1	120	2.0828	69	1.8451	18	1.2788
	Mean		2.0499		1.5377		1.2788
B	SDC-4-2	1,207	3.0821	1,971	3.2949	686	2.8370
B	SDC-7-2	1,519	3.1818	1,268	3.1035	869	2.9395
	Mean		3.1319		3.1992		2.8883
C	SDC-4-3	4,466	3.6500	5,267	3.7216	5,835	3.7661
C	SDC-7-5	6,587	3.8188	9,432	3.9746	7,848	3.8948
	Mean		3.7344		3.8481		3.8305

Inner - Year 2 (grid survey)

Zone	Station	July/71 x	July/71 y	Nov./71 x	Nov./71 y	April/72 x	April/72 y
A	NDC-.5-1	815	2.9117	72	1.8633	18	1.2788
A	SDC-.5-1	253	2.4048	162	2.2122	1,177	3.0711
	Mean		2.6583		2.0377		2.1750
B	NDC-1-2	1,793	3.2358	2,862	3.4568	325	2.5132
B	SDC-1-2	9,244	3.9659	741	2.8704	3,424	3.5347
	Mean		3.6009		3.1636		3.0239
C	NDC-1-3	2,808	3.4486	7,376	3.8679	959	2.9823
C	SDC-1-3	1,793	3.2538	18,708	4.2721	5,128	3.7100
	Mean		3.3422		4.0700		3.3462

(cont.)

TABLE 5 (cont.)

Outer - Year 2 (grid survey)

Zone	Station	July/71 x	July/71 y	Nov./71 x	Nov./71 y	April/72 x	April/72 y
A	SDC-4-1	416[a]	2.6201	144	2.1614	344	2.5378
A	SDC-7-1	543	2.7356	0[b]	0.0000	326	2.5145
	Mean		2.6778		1.0807		2.5262
B	SDC-4-2	1,448	3.1611	397	2.5999	777	2.8910
B	SDC-7-2	869	2.9395	543[c]	2.7356	578	2.7627
	Mean		3.0503		2.6677		2.8269
C	SDC-4-3	12,906	4.1108	8,554	3.9322	6,487	3.8121
C	SDC-7-5	10,241	4.0104	506[d]	2.7050	10,875	4.0365
	Mean		4.0606		3.3186		3.9243

[a]July/71: no data for SDC-4-1. The count for NDC-4-1 was used instead.
[b]Nov./71; SDC-7-1: no data. Used NDC-4-1.
[c]Nov./71; SDC-7-2: no data. Used NDC-4-2.
[d]Nov./71; SDC-7-5: no data. Used NDC-4-3.

Inner - Year 3[e] (random survey)

Zone	Station	July/72 x	July/72 y	Oct./72 x	Oct./72 y	April/73 x	April/73 y
A	D01,04,07[f]	6,038	3.7810	2,285	3.3591	0	0.0000
A	D02,05,08	734	2.8663	4,162	3.6194	7,344	3.8660
A	D03,06,09	143	2.1584	20	1.3222	20	1.3222
	Mean		2.9352		2.7669		1.7294
B	D11,14,17	1,293	3.1119	930	2.9869	2,828	3.4516
B	D12,15,18	14,200	4.1523	969	2.9868	3,050	3.4844
B	D13,16,19	4,566	3.6596	6,322	3.8009	3,171	3.5013
	Mean		3.6413		3.2522		3.4791
C	D21,24,27	10,544	4.0230	7,797	3.8920	20,382	4.3093
C	D22,25,28	19,028	4.2794	18,281	4.2620	3,515	3.5460
C	D23,26,29	1,838	3.2646	17,008	4.2307	4,667	3.6691
	Mean		3.8557		4.1282		3.8415

[e]See footnote g, next page.

[f]This notation means that D01 was used in July/72, D04 was used in Oct./72, and D07 was used in April/73. For the meaning of the station names, see Mozley (1973).

(cont.)

TABLE 5 (cont.)

		Outer - Year 3[g] (random survey)					
		July/72		Oct./72		April/73	
Zone	Station	x	y	x	y	x	y
A	S01,04,07	571	2.7574	122	2.0899	204	2.3118
A	S02,05,08	1,306	3.1163	1,306	3.1163	2,652	3.4237
A	S03,06,09	836	2.9227	224	2.3522	61	1.7924
	Mean		2.9321		2.5195		2.5093
B	S11,14,17	2,728	3.4360	1,658	3.2198	61,570	4.7894
B	S12,15,18	5,293	3.7238	2,485	3.3955	7,576	3.8795
B	S13,16,19	16,988	4.2302	3,030	3.4816	3,312	3.5202
	Mean		3.7967		3.3656		4.0630
C	S21,24,27	5,494	3.7400	8,848	3.9469	14,665	4.1663
C	S22,25,28	13,393	4.1269	24,159	4.3831	32,704	4.5146
C	S23,26,29	27,391	4.4376	19,028	4.2794	24,826	4.3949
	Mean		4.1015		4.2031		4.3586

[g] The data of Year 3 were subjected to analysis of variance to estimate th sampling error of the random survey (Table 4). The input to that ANOVA consisted of the transformed densities at the individual stations, rather than the zone means of the transformed densities as in the ANOVA of Table 8.

It is worth considering whether the data obtained using the grid survey (July/70 through April/72) show an error variance that is similar to the value just obtained from the random survey results. An answer can be obtained by dividing the grid into depth zones and choosing inner and outer groups of stations within each depth range. Such a grouping was described in Part 18. Table 6 presents a slightly different grouping, using 30 of the 46 stations. This arrangement puts an equal number of stations in each group and avoids those stations for which there was a lot of missing data in the months being studied. The five inner stations needed for each zone were

TABLE 6. *Data for the Estimate of Sampling Error in the Grid Survey.*

Columns headed by "x" contain the actual densities in animals per square meter. Columns headed by "y" contain the corresponding transformed values. The transformation is y = log(x+1). "ND" means "no data". See Table 3 for the analysis of variance of this data. The five replicates within each cell are the densities at five individual stations, selected from those stations in the grid that lie within the given depth range and have the appropriate outfall distance (inner or outer). The inner stations are those that lie on one of the seven innermost transects (SDC-1, SDC-.5, DC, NDC-.5, NDC-1); the others are classified as outer. The sample at each station was the combined contents of two grabs. The data given here previously appeared in Table 10 of Ayers et al. (1971) or in Tables 38-44 of Mozley (1973).

Inner - 1970

		July		Sept.		Nov.	
Zone	Station	x	y	x	y	x	y
A	DC-1	962	2.9836	ND	ND	ND	ND
A	NDC-.5-1	139	2.1461	372	2.5717	50	1.7076
A	SDC-.5-1	226	2.3560	1,727	3.2375	893	2.9513
A	NDC-1-1	521	2.7168	502	2.7016	416	2.6201
A	SDC-1-1	278	2.4440	77	1.8921	2,268	3.3558
B	DC-2	2,519	3.4014	2,493	3.3969	3,640	3.5612
B	NDC-.5-2	ND	ND	2,840	3.4535	2,371	3.3751
B	SDC-.5-2	1,023	3.0102	1,260	3.1007	1,850	3.2674
B	NDC-1-2	3,006	3.4781	27,681	4.4422	1,363	3.1348
B	SDC-1-2	4,980	3.6973	3,025	3.4809	2,483	3.3952
C	DC-3	7,319	3.8645	5,675	3.7540	2,465	3.3920
C	NDC-.5-3	5,875	3.7691	2,450	3.3893	6,761	3.8301
C	SDC-.5-3	5,154	3.7122	4,544	3.6575	3,042	3.4833
C	NDC-1-3	10,084	4.0037	17,465	4.2422	4,241	3.6276
C	SDC-1-3	5,755	3.7601	17,379	4.2400	17,326	4.2387

(cont.)

TABLE 6 (cont.)

<u>Inner - 1971</u>

Zone	Station	July x	July y	Sept. x	Sept. y	Nov. x	Nov. y
A	DC-1	ND	ND	ND	ND	126	2.1038
A	NDC-.5-1	815	2.9117	1,159	3.0645	72	1.8633
A	SDC-.5-1	253	2.4048	1,703	3.2315	162	2.2122
A	NDC-1-1	1,232	3.0910	634	2.8028	0	0.0000
A	SDC-1-1	869	2.9395	815	2.9117	162	2.2122
B	DC-2	3,623	3.5592	4,130	3.6161	1,921	3.2838
B	NDC-.5-2	2,736	3.4373	3,334	3.5231	597	2.7767
B	SDC-.5-2	1,829	3.2625	2,971	3.4730	2,752	3.4398
B	NDC-1-2	1,793	3.2538	1,593	3.2025	2,862	3.4568
B	SDC-1-2	9,244	3.9659	7,521	3.8763	741	2.8704
C	DC-3	1,032	3.0141	ND	ND	2,138	3.3302
C	NDC-.5-3	7,285	3.8625	6,362	3.8037	19,395	4.2877
C	SDC-.5-3	2,573	3.4106	8,410	3.9248	1,593	3.2025
C	NDC-1-3	2,808	3.4486	10,985	4.0408	7,376	3.8679
C	SDC-1-3	1,793	3.2538	12,652	4.1022	18,708	4.2721

<u>Outer - 1970</u>

Zone	Station	July x	July y	Sept. x	Sept. y	Nov. x	Nov. y
A	NDC-2-1	0	0.0000	696	2.8432	877	2.9435
A	SDC-2-1	677	2.8312	138	2.1430	138	2.1430
A	NDC-2-2	ND	ND	1,040	3.0175	3,669	3.5647
A	NDC-4-1	295	2.4713	60	1.7853	68	1.8388
A	SDC-4-1	103	2.0170	354	2.5502	16	1.2304
B	SDC-2-2	1,268	3.1035	1,780	3.2507	2,502	3.3985
B	NDC-2-3	1,319	3.1206	1,385	3.1418	7,762	3.8900
B	SDC-2-3	10,509	4.0216	8,953	3.9520	16,961	4.2295
B	NDC-4-2	1,172	3.0693	172	2.2380	686	2.8370
B	SDC-4-2	1,207	3.0821	1,475	3.1691	1,971	3.2949
C	NDC-2-4	16,789	4.2251	6,363	3.8037	18,248	4.2612
C	SDC-2-4	9,118	3.9599	12,657	4.1024	519	2.7160
C	NDC-4-3	2,128	3.3282	1,041	3.0179	4,033	3.6057
C	SDC-4-3	4,466	3.6500	15,335	4.1857	5,267	3.7216
C	SDC-7-4	6,554	3.8166	26,716	4.4268	2,945	3.4692

(cont.)

TABLE 6 (cont.)

Outer - 1971

Zone	Stations	July x	July y	Sept. x	Sept. y	Nov. x	Nov. y
A	NDC-2-1	561	2.7497	290	2.4639	90	1.9590
A	SDC-2-1	416	2.6201	217	2.3385	543	2.7356
A	NDC-2-2	1,249	3.0969	706	2.8494	778	2.8915
A	NDC-4-1	416	2.6201	289	2.4624	0	0.0000
A	SDC-4-1	ND	ND	181	2.2601	144	2.1614
B	SDC-2-2	2,445	3.3884	2,825	3.4512	1,085	3.0358
B	NDC-2-3	5,311	3.7253	1,377	3.1392	740	2.8698
B	SDC-2-3	1,576	3.1978	5,582	3.7469	3,822	3.5824
B	NDC-4-2	144	2.1614	343	2.5366	543	2.7356
B	SDC-4-2	1,448	3.1611	2,717	3.4342	397	2.5999
C	NDC-2-4	29,912	4.4759	28,154	4.4496	11,837	4.0733
C	SDC-2-4	15,155	4.1806	15,862	4.2004	15,099	4.1790
C	NDC-4-3	1,684	3.2266	4,622	3.6649	506	2.7050
C	SDC-4-3	12,906	4.1108	10,458	4.0195	8,554	3.9322
C	SDC-7-4	14,611	4.1647	4,441	3.6476	ND	ND

chosen from the available stations that lay on the seven innermost transects (SDC-1, SDC-.5, DC, NDC-.5, NDC-1). The five outer stations of each zone were chosen from the remaining six transects. The stations in each group provide five replicate values for each cell of a four-way analysis of variance. The sample taken at each station was the combined contents of two grabs. (It would have been preferable if the two grabs had been counted separately.) To make the error variance of these data comparable to that obtained from the random survey, it is necessary to multiply it by two. (The variance of the mean of two observations drawn at random from some distribution is half the variance of the distribution itself).

The results of the four-way ANOVA are shown in Table 7. Note that the error variance is 0.28464. When this is multiplied by two it becomes 0.56928, which is quite comparable to the value obtained from the random survey, 0.49527. However, the latter value is the one we will use in later calculations; it will be referred to as the sampling error.

Estimation of the Interaction Variance $\sigma^2_{\beta\gamma}$

To obtain a value for $\sigma^2_{\beta\gamma}$, data from a number of years must be considered. Table 2 shows the surveys that are available for this purpose. The more years that can be used, the better the estimate. It is also desirable to include as many seasons as possible from each year. The arrangement we have chosen is as follows year 1--July/70, Nov./70, April/71; year 2--July/71, Nov./71, April/72; year 3--July/72, Oct./72, April/73. All data in years 1 and 2 were taken with the grid survey; the data in year 3 were taken with the random survey.

The fact that both grid and random data were to be considered in one ANOVA caused some difficulties. First, the areas sampled by the "outer" station groups are not the same (though they are remote from the outfall in both cases); second, the individual data values are not exactly comparable. The latter is because the datum at each station of the grid survey was the mean of two grabs. The grabs were physically combined before counting, so the individual grab counts are unavailable. Moreover, the random survey data consist of the results of three one-third grab casts at each station. The mean of these three may or may not be comparable to what would have been obtained with one cast of a whole grab, since the results from the three chambers might be correlated.

This situation was handled with a compromise method. We computed the

TABLE 7. *Estimation of the Sampling Error in the Grid Survey.*

The method used was a four-way factorial analysis of variance of six selected grid surveys. The data were total animals per meter^2, transformed through $y = \log(x+1)$.

Factors:

1. Outfall distance(2): inner, outer
2. Year(2): 1970, 1971
3. Depth zone(3): A (0-8m), B (8-16m), C (16-24m)
4. Month(3): July, September, November

Within each cell, five individual stations were used as replicates. The sample taken at each station was the combined contents of two grabs.

Source of Variation	Degrees of freedom	Sums of squares	Mean squares	F-ratio
1. Outfall distances	1	.64175	.64175	2.56
2. Years	1	.00446	.00446	.02
3. Depth zones	2	54.44033	27.22017	95.63**
4. Months	2	2.24850	1.12425	3.95
12	1	.14829	.14829	.52
13	2	.85552	.42776	1.50
14	2	.19843	.09922	.35
23	2	.25941	.12970	.46
24	2	1.95730	.97865	3.44*
34	4	.74026	.18507	.65
123	2	.41964	.20982	.74
124	2	.33970	.16985	.60
134	4	.86831	.21708	.76
234	4	2.86123	.71531	2.51*
1234	4	.94894	.23723	.83
Error	135#	38.42679	.28464	
Total	170	105.35886		

* Significant, $p<.05$
** Significant, $p<.01$

$F_{.05(1,125)} = 3.92$, $F_{.05(2,125)} = 3.07$, $F_{.05(4,125)} = 2.44$

#The numerator sums of squares and the error degrees of freedom have been adjusted for the nine missing data values by the method of Winer (1971, p.402).

zone means for each type of survey, and ran an ANOVA on the means rather than on the individual station values. This method is unable to give a value for the error variance, but we already have an estimate of that from the preceding section. The zone means for the grid survey were obtained using two stations, so that each datum is the mean of four grabs. The zone means for the random survey were obtained using three stations, so that each datum is either the mean of three whole grabs (zone 0) or the mean of nine one-third grabs (zones 1 and 2). Ideally we might have performed some kind of weighted-means analysis, since the area sampled is not exactly the same in all cells, but it was decided to do a regular ANOVA. The layout and results of this appear in Table 8. The data that were analyzed are in Table 5.

As before, Table 8 includes the results of the usual significance tests, but they are not of prime interest. The quantity we are concerned about is the mean square for the interaction of year and outfall distance. MS = .255085 for that interaction. To interpret the value, it is necessary to write down a model equation for the appropriate ANOVA and derive some expected mean squares.

At this point a complication occurs, since we want to be able to compare $\hat{\sigma}_{\beta\gamma}$ with the previously obtained sampling error. (The notation $\hat{\sigma}_{\beta\gamma}$ means "an estimate of $\sigma_{\beta\gamma}$".) It is necessary to write down an ANOVA design that includes three-fold replication even though the analysis given in Table 8 does not. The mean squares in Table 8 can be corrected to the values they would have assumed with three-fold replication by multiplying each of them by three. This can be justified by noting that all sum of squares formulas for a factorial ANOVA, except the total and the error sums of squares, involve summing first over replicates before doing any squaring or any other summations, and that such formulas also require dividing through by

TABLE 8. *Estimation of the Interaction Variance* $\sigma^2_{\beta\gamma}$. The method used was a four-way mixed model factorial analysis of variance of nine surveys, six grid and three random. (See Table 5 for the data analyzed.) The quantity analyzed was total animals per square meter, transformed through $y = \log(x+1)$. The entry in each cell was the zone mean of the transformed variable, so there was in effect just one replicate per cell. Note the column in the table that indicates which mean square was used as the denominator of F. Table 9 provides the list of expected

Factor	Name	No. of Levels	Type
Y	Year	3	random
D	Outfall distance	2	fixed
Z	Depth zone	3	fixed
S	Season	3	fixed

Source of Variation	df	Sums of Squares	Mean Squares	Denom. of F	F-ratio
Year	2	2.61669	1.30835	YDZS	18.13**
Outfall distance	1	.00996	.00996	YD	.04
Depth zone	2	23.41074	11.70537	YZ	215.53**
Season	2	.98412	.49206	YS	3.08
Year x outfall distance	2	.51017	.25508	YDZS	3.53
Year x zone	4	.21724	.05431	YDZS	.75
Year x season	4	.64002	.16000	YDZS	2.22
Outfall distance x zone	2	.42986	.21493	YDZ	2.27
Outfall distance x season	2	.96634	.48317	YDS	6.61
Zone x season	4	.62625	.15656	YZS	1.20
Y x D x Z	4	.37900	.09475	YDZS	1.31
Y x D x S	4	.29222	.07305	YDZS	1.01
Y x Z x S	8	1.04335	.13402	YDZS	1.81
D x Z x S	4	.26505	.06626	YDZS	.92
Y x D x Z x S	8	.57740	.07217	--	--
Total	53	32.96841			

$F_{.05(1,2)} = 18.51$ $F_{.05(2,8)} = 4.46$ $F_{.01(2,8)} = 8.65$

$F_{.05(2,4)} = 6.94$ $F_{.05(4,8)} = 3.84$

$F_{.01(2,4)} = 18.00$ $F_{.05(8,8)} = 3.44$

the number of replicates. Thus, if two ANOVAs have identical cell means, and one has 1 replicate while the other has n, then the mean squares of the second are larger by a factor n^2/n, or n. The exception is the error MS, which the first ANOVA lacks because it is unreplicated.

A suitable three-fold replicated analog to the Table 8 ANOVA is shown in Table 9, and is designated as Design B. This is a mixed model, since the year factor is random and the other three are fixed. We make year a random factor because we are not interested in the three individual year means but in the population of years from which those three are a sample. We use a model with three replicates for convenience, although the means analyzed in Table 8 are sometimes based on the area of three grabs and sometimes on the area of four grabs, as discussed earlier.

Table 9 contains a line from which $\sigma^2_{\beta\gamma}$ can be estimated. It is the Y x D interaction (year x outfall distance), whose expected mean square is $\sigma_\epsilon^2 + 27\,\sigma^2_{\beta\gamma}$. The MS for year x outfall distance in Table 8 is .255085. This should be multiplied by three as discussed above; the result is .76526. The sampling error, σ_ϵ^2 , was estimated in the last section as .49527. Hence, an estimate of $\sigma^2_{\beta\gamma}$ is (.76526 - .49527)/27 = .01000.

Although it is permissible to estimate $\sigma^2_{\beta\gamma}$ in this way, it should be noted that zero in another reasonable estimate. Let us test the null hypothesis that $\sigma^2_{\beta\gamma} = 0$. If this is true, then the two quantities found above, .76526 and .49527, would be estimates of the same population variance σ_ϵ^2 . The first of these comes from a mean square with 2 d.f., the second from a mean square with 36 d.f. $F_s = .76526/.49527 = 1.55$. From a table, $F_{.05}\,(2,36) = 3.26$. Thus, the hypothesis that $\sigma^2_{\beta\gamma} = 0$ cannot be rejected using the available data. On the other hand, the hypothesis that $\sigma^2_{\beta\gamma}$ = .01000 cannot be rejected either. If this were true, then we would have:

TABLE 9. *Linear Model and Expected Mean Squares for Design B, a Factorial Analysis of Variance.*

$$X_{jkmpq} = \mu + \beta_j + \gamma_k + \delta_m + \zeta_p + \beta\gamma_{jk} + \beta\delta_{jm} + \beta\zeta_{jp} + \gamma\delta_{km} + \gamma\zeta_{kp} + \delta\zeta_{mp} + \beta\gamma\delta_{jkm} + \beta\gamma\zeta_{jkp} + \beta\delta\zeta_{jmp} + \gamma\delta\zeta_{kmp} + \beta\gamma\delta\zeta_{jkmp} + \varepsilon_{q(jkmp)}$$

Factor	Greek Symbol	Name	No. of Levels	Type
Y	β	Year	3	random
D	γ	Outfall distance	2	fixed
S	δ	Season	3	fixed
Z	ζ	Depth zone	3	fixed

This is a mixed model ANOVA, since factor Y is random while the others are fixed.

Source of variation	Degrees of freedom	Expected mean square
Y	2	$\sigma_\varepsilon^2 + 54\sigma_\beta^2$
D	1	$\sigma_\varepsilon^2 + 81\theta_\gamma^2 + 27\sigma_{\beta\gamma}^2$
S	2	$\sigma_\varepsilon^2 + 54\theta_\delta^2 + 18\sigma_{\beta\delta}^2$
Z	2	$\sigma_\varepsilon^2 + 54\theta_\zeta^2 + 18\sigma_{\beta\zeta}^2$
YD	2	$\sigma_\varepsilon^2 + 27\sigma_{\beta\gamma}^2$
YS	4	$\sigma_\varepsilon^2 + 18\sigma_{\beta\delta}^2$
YZ	4	$\sigma_\varepsilon^2 + 18\sigma_{\beta\zeta}^2$
DS	2	$\sigma_\varepsilon^2 + 27\theta_{\gamma\delta}^2 + 9\sigma_{\beta\gamma\delta}^2$
DZ	2	$\sigma_\varepsilon^2 + 27\theta_{\gamma\zeta}^2 + 9\sigma_{\beta\gamma\delta}^2$
SZ	4	$\sigma_\varepsilon^2 + 18\theta_{\delta\zeta}^2 + 6\sigma_{\beta\delta\zeta}^2$

TABLE 9 (cont.)

Source of Variation	Degrees of Freedom	Expected Mean Square
YDS	4	$\sigma_\epsilon^2 + 9\,\sigma^2_{\beta\gamma\delta}$
YDZ	4	$\sigma_\epsilon^2 + 9\,\sigma^2_{\beta\gamma\zeta}$
YSZ	8	$\sigma_\epsilon^2 + 6\,\sigma^2_{\beta\delta\zeta}$
DSZ	4	$\sigma_\epsilon^2 + 9\,\theta^2_{\gamma\delta\zeta} + 3\,\sigma^2_{\beta\gamma\delta\zeta}$
YDSZ	8	$\sigma_\epsilon^2 + 3\,\sigma^2_{\beta\gamma\delta\zeta}$
Error	see text	σ_ϵ^2

$$MS_{YD} = .76526 = \sigma_\epsilon^2 + 27\ (.01000) = \sigma_\epsilon^2 + .27000\ .$$

This can be solved to give an estimate of .49526 for σ_ϵ^2. F_s = .49526/.49527= 1.00, which is not significant. We will use the estimate $\hat{\sigma}^2_{\beta\gamma}$ = .01000. The present estimate of $\sigma^2_{\beta\gamma}$ is based on only 2 degrees of freedom (the number of years of data minus one). When more years of data are available, its value can be assigned with more confidence. To be prepared for the possibility that it is non-zero is the more conservative course, and it is the one that we favor.

Justification for a non-zero estimate of an added variance component in a case where it is not significant is given by Sokal and Rohlf (1969, p. 265).

Calculation of the Least Detectable True Change δ

It is now possible to calculate the smallest true change detectable using the eight-year design (Design A), with a 5% significance level and 95%

power. The error standard deviation needed was previously found to be $\sigma = \sqrt{\sigma_\varepsilon^2 + 27\,\sigma_{\beta\gamma}^2}$. Estimates of the two quantities under the radical sign are now available: $\hat{\sigma}_\varepsilon^2 = .49527$, based on 36 degrees of freedom, and $\hat{\sigma}_{\beta\gamma}^2 = .01000$, based on 2 degrees of freedom. These lead to $\hat{\sigma} = \sqrt{.76526} = .87479$.

A further step is to compute a δ value from this σ . A quantity δ was presented earlier as a least detectable true difference where two treatment means in an ANOVA were to be compared. In the present case we are computing the power of an F-test of an interaction, and some algebra is required to show how the δ given by Sokal and Rohlf's equation can be applied here.

The first step is to present some equations of Cohen (1969). Using his own measure for the root mean square amplitude of a true interaction effect, σ_x , he gives the following equation (p. 365):

$$\sigma_x = \sqrt{\frac{\sum_{1}^{rc} x_{ij}^2}{(r-1)(c-1)+1}}$$

This applies to a two-way interaction in a two-factor analysis of variance, having r rows (levels of the first factor) and c columns (levels of the second factor). x_{ij} is the interaction effect in cell ij, and it is given by: $x_{ij} = m_{ij} - m_{i.} - m_{.j} + m$ (m_{ij} is the population mean [true mean] of cell ij; $m_{i.}$ is that of row i; $m_{.j}$ is that of column j; m is the grand population mean). In his notation $\sigma_x = f\sigma$, where f = d/2 for a comparison involving a single degree of freedom (p. 269). His d is identical with our δ/σ, so σ_x is just $\delta/2$. The formula for σ_x given above can be rewritten in our own notation as:

$$\delta = 2\sqrt{\frac{\sum_{1}^{rc} x_{ij}^2}{df+1}}$$

where df are the degrees of freedom for the interaction.

The next question is: how can the above formula for a two-factor ANOVA be applied in the present case (i.e. in Design A) which uses a five-factor ANOVA? The answer is, that for calculating x_{ij} and the number of observations per cell (n) the other three factors should be "collapsed" so that a two-factor ANOVA results. Design A is a 2 x 4 x 2 x 3 x 3 problem, with 3 replicates, so that there are 144 cells and a total of 432 observations. By collapsing the month, zone and year factors we are left with a two-factor problem with four cells: construction time (2) x outfall distance (2). The mean of each cell, when the complete data are available in 1979, will be the mean of the transformed animal densities over all months, zones and years for stations with the appropriate outfall distance and construction time. Since 432 observations will be divided equally into four groups, there will be a total of 108 observations per cell. According to Cohen, "... the n which governs the power of an R x C interaction test is the cell n ..." (p. 367). Hence we set n = 108 in the Sokal and Rohlf formula for δ .

Derivation of a Least Detectable True Ratio in Terms of δ

What is the relationship of the four x_{ij} values in the collapsed 2 x 2 ANOVA? Since interaction effects must sum to zero in each row and column, there is only a single degree of freedom in this case. Each interaction must equal either x or -x, where x is the common absolute value of the four interactions. Thus $\sum_{1}^{rc} x_{ij}^2 = 4x^2$. Putting df = 1 in the Cohen formula

presented earlier, we obtain $\delta = 2\sqrt{4x^2/2} = 2\sqrt{2}\,x$. Thus $x = \delta/2\sqrt{2}$.

The next step is to relate δ to the untransformed animal densities. Let the value of the true row main effect in the first row be f; let the value of the true column main effect in the first column be g; m is the true grand mean. The number of animals per square meter in each cell is then as shown below:

	Inner	Outer
Before	$I_B = 10^{m+f+g-x} - 1$	$O_B = 10^{m+f-g+x} - 1$
After	$I_A = 10^{m-f+g+x} - 1$	$O_A = 10^{m-f-g-x} - 1$

I_B is the true mean number of animals per square meter at the inner stations before plant operation; I_A is the mean at the inner stations after operation starts; O_B is the mean at the outer stations before operation; O_A is the mean at the outer stations after operation starts. Ratios involving those mean numbers of animals can be set up as follows.

$$\frac{I_B + 1}{O_B + 1} = \frac{10^{m+f+g-x}}{10^{m+f-g+x}} = 10^{2g-2x}$$

$$\frac{I_A + 1}{O_A + 1} = \frac{10^{m-f+g+x}}{10^{m-f-g-x}} = 10^{2g+2x}$$

In our design we want to compare the preoperational ratio with the operational ratio. We do so by forming a new ratio R:

$$R = \left(\frac{I_A + 1}{O_A + 1}\right) \Big/ \left(\frac{I_B + 1}{O_B + 1}\right) = \frac{10^{2g+2x}}{10^{2g-2x}} = 10^{4x}$$

It was found earlier that $x = \delta/2\sqrt{2}$ when it is just detectable with P = .95.

Thus, if R is to be detectable it must obey the following inequality:

$$R \geq 10^{\sqrt{2}\delta}$$

δ was previously called the least detectable true difference of the transformed variable. We now designate the quantity $10^{\sqrt{2}\delta}$ as the "least detectable true ratio." The test from which the Sokal and Rohlf formula comes is two-tailed. This means that a true population change is detectable with P = .95 either if $R \geq 10^{\sqrt{2}\delta}$ or $R \leq 10^{-\sqrt{2}\delta}$. (Our test can pick up either a relative increase or a relative decrease of the inner populations.) To find a numerical estimate of R for the benthos survey, we now obtain a value for δ.

Evaluation of δ Using the Variance Determined Previously

Values required by the Sokal and Rohlf formula for δ that have been determined in the preceding sections are: α = .05, P = .95, $\hat{\sigma}$ = .87479 and n = 108. The quantity still needed is ν, the degree of freedom for the denominator of the F-test. Table 1 indicates that ν = 6 for our chosen denominator, which is MS_{YD}. It was previously mentioned that the numerator, MS_{CD}, has a single degree of freedom. If it had more than one, then the Sokal and Rohlf formula could not be used. The five relevant quantities can now be substituted in the formula:

$$\delta = \sigma\sqrt{\frac{2}{n}}\left(t_{\alpha[\nu]} + t_{2(1-P)[\nu]}\right)$$

$$= .87479\sqrt{\frac{2}{108}}\left(t_{.05[6]} + t_{.10[6]}\right)$$

$$= .87479\,(.136083)\,(2.447 + 1.943)$$

$$\delta = .52260$$

$$10^{\sqrt{2}\delta} = 5.484; \quad 10^{-\sqrt{2}\delta} = .182$$

Thus, the least detectable true ratio R = 5.484. Benthos at the inner stations must undergo a true 5.5-fold increase or decrease relative to the outer stations in order to be detected by the eight-year survey.

REFERENCES

Ayers, J. C., W. L. Yocum, H. K. Soo, T. W. Bottrell, S. C. Mozley and L. C. Garcia (1971): Benton Harbor Power Plant Limnological Studies, part 9 "The biological survey of 10 July 1970." Great Lakes Research Division, University of Michigan, Ann Arbor, Michigan.

Beak Consultants (1973): "Evaluation of pre-operational biological conditions of Lake Michigan in the vicinity of the Palisades nuclear plant." A report to Consumers Power Co., Jackson, Michigan, March 2, 1973.

Cohen, Jacob (1969): Statistical Power Analysis for the Behavioral Sciences. Academic Press, New York. 415 p.

Elliott, J. M. (1971): Some Methods for the Statistical Analysis of Samples of Benthic Invertebrates. Freshwater Biological Association, Scientific Publication No. 25, 144 p.

Johnston, Edward M. (1973): "Effect of a thermal discharge on benthos populations: statistical methods for assessing the impact of the Cook nuclear plant." Benton Harbor Power Plant Limnological Studies, part 18, Great Lakes Research Division, University of Michigan, Ann Arbor, Michigan 20 p.

Kirk, Roger E. (1968): Experimental Design: Procedures for the Behavioral Sciences. Brooks/Cole, Belmont, California. 577 p.

Mozley, Samuel C. (1973): "Study of benthic prganisms." In J. C. Ayers and E. Seibel (ed.), Benton Harbor Power Plant Limnological Studies, part 13, "Cook Plant pre-operational studies 1972," p. 178-242. Great Lakes Research Division, University of Michigan, Ann Arbor, Michigan.

Myers, Jerome L. (1972): Fundamentals of Experimental Design. Second edition. Allyn and Bacon, Boston. 465 p.

Scheffé, Henry (1959): The Analysis of Variance. Wiley, New York, 477 p.

Sokal, Robert R. and F. James Rohlf (1969): Biometry. W. H. Freeman, San Francisco. 776 p.

Winer, B. J. (1971): Statistical Principles in Experimental Design. Second edition. McGraw-Hill, New York.